Hysteresis and magnetization jumps in the $T=0$ dynamics of spin glasses

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We present results from Monte Carlo simulations of hysteresis in the zero-temperature ($T=0$) dynamics of the Sherrington-Kirkpatrick spin glass model. We study the statistics of *magnetization-jumps* (denoted as Δm) in response to a time-dependent magnetic field $H(t)$, which increases or decreases with constant increments Δ as $H(t) \rightarrow H(t) \pm \Delta$. In particular, we focus on the field dependence of the Δm-distribution function $P(\Delta m, H)$. We formulate arguments to understand the variation of $P(\Delta m, H)$ along the hysteresis loop in the weak-disorder limit.

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I. INTRODUCTION

An important nonequilibrium property of magnetic materials is *hysteresis*, i.e., the delayed response of a spin system to a time-dependent magnetic field [1]. In many experimental situations, the magnetic field is periodic in time with $H(t+t_0)=H(t)$, where t_0 is the time period. The plot of magnetization per spin $m(t)$ vs $H(t)$ is referred to as the *hysteresis loop*. In pure ferromagnetic systems, hysteresis occurs because of the existence of metastable states in the vicinity of a first-order phase transition. For example, the down state is metastable for $H_s(T) > H > 0$, where the metastability limit $H_s(T)$ depends on the temperature T. The timescale of transition to the stable up state τ_s depends on H, T, and the intrinsic spin-flip timescale τ_i. Thus the system response is determined by the competition between the experimental timescale (t_0) and τ_s. In the limit $\tau_s/t_0 \rightarrow 0$, the spins have enough time to align with the external field and the magnetization exhibits a discontinuity at $H=0$. In this case, the "hysteresis loop" is a step function. When $\tau_s/t_0 \sim O(1)$, the field changes appreciably before the spins readjust, leading to well-defined hysteresis loops. Finally, in the limit $\tau_s/t_0 \rightarrow \infty$, the spins cannot respond to the field and no hysteresis loops are observed at all.

The $T=0$ limit is relevant in many experimental situations [2,3] where the barriers to domain flipping are too large to be overcome by fluctuations. Consider the two-state Ising model for an N-spin system $\{S_i\}$:

$$\mathcal{H} = -J\sum_{\langle ij \rangle} S_i S_j - H(t)\sum_i S_i, \quad S_i = \pm 1, \tag{1}$$

where J is the exchange coupling, and $\langle ij \rangle$ denotes a sum over nearest-neighbor pairs. For $H(t) < 0$, all spins are aligned in the stable state $S_i = -1$. As $H(t)$ is increased, this state continues to be metastable for $H_s(T=0) = zJ > H(t) > 0$, where z is the *coordination number*. At $T=0$, the timescale for escape from the metastable state is $\tau_s = \infty$. The local field at each site, $h_i = H(t) - zJ$, becomes positive for $H(t) > zJ$ and the spin flips into the up state spontaneously, i.e., with $\tau_s = 0$ assuming that the intrinsic timescale $\tau_i \ll t_0$. The opposite scenario arises when the field $H(t)$ changes from $H > 0$ to $H < 0$. Thus the resultant hysteresis loop is rectangular, with discontinuities in m at $H = \pm zJ$.

Of course, real ferromagnets are not pure and often contain impurities. Let us focus on the case of quenched disorder, which corresponds to the case of immobile impurities. The free-energy landscape is drastically modified by the presence of disorder, and is characterized by a large number of metastable minima with a distribution of barrier heights [4,5]. Consider the $T=0$ hysteresis of disordered systems, where there are no fluctuation-induced transitions across free-energy barriers. Transitions occur only when the change in magnetic field eliminates a metastable state where the system is trapped. The system then evolves to a neighboring metastable state, and so on. Thus the rectangular loop of the disorder-free magnet is replaced by a smoother loop consisting of a series of magnetization jumps or avalanches, which are referred to as *Barkhausen noise* [6,7]. Apart from ferromagnetic materials, a wide variety of experimental systems such as martensites [8] and superconducting films [9] exhibit related phenomena.

In this paper, we focus on hysteresis and magnetization-jump distributions in the $T=0$ dynamics of spin glasses. In Sec. II, we review some studies of $T=0$ hysteresis in model disordered systems. In Sec. III, we present numerical results obtained from Monte Carlo (MC) simulations of hysteresis in the Sherrington-Kirkpatrick (SK) spin glass [10]. In particular, we study magnetization-jump distributions along the hysteresis loop. In Sec. III, we also present arguments to understand the nature of these distributions at different H values. Finally, Sec. IV concludes this paper with a summary.

II. OVERVIEW OF RELEVANT RESULTS

Consider the Ising Hamiltonian with quenched disorder and a time-dependent external field:

$$\mathcal{H} = -\sum_{i>j} J_{ij} S_i S_j - \sum_{i=1}^{N} [H(t) + H_i] S_i, \quad S_i = \pm 1, \tag{2}$$

where we have introduced disorder via (a) random exchange couplings or bonds J_{ij}, and (b) a random magnetic field H_i. In Eq. (2), we have allowed for the possibility of long-ranged exchange interactions. The Ising Hamiltonian does not have intrinsic dynamics so we associate stochastic kinetics by placing it in contact with a heat bath [11]. The appropriate stochastic kinetics for the ferromagnet is Glauber spin-flip kinetics, where an arbitrary spin is flipped as $S_i \to -S_i$. For $T=0$ dynamics, the spin flip is accepted only if the system energy decreases. We consider a situation where $H(t)$ is changed slowly from $-\infty \to \infty$ ($S_i = -1 \to +1 \ \forall \ i$) or $\infty \to -\infty (S_i = +1 \to -1 \ \forall \ i)$.

Let us first discuss the case of the nearest-neighbor random-field Ising model (RFIM), where $J_{ij}=J$ when i, j are nearest neighbors, and 0 otherwise. An important study of the $T=0$ dynamics is due to Sethna *et al.* [12]. In their study, the magnetic field was changed adiabatically, i.e., till the occurrence of the first spin-flip. They clarified the following properties of hysteresis loops in the RFIM:

(1) The loops exhibit *return-point memory* (RPM). Assume that the increasing field $H(t)$ is stopped at a value H_2, and then decreased to H_1. Now, if the field is again increased to H_2, the system returns to precisely the same state as before. The RPM property holds for all sub-loops, sub-sub-loops, etc., in the original hysteresis loop.

(2) The hysteresis loops are characterized by avalanches, corresponding to transitions between metastable states in the energy landscape. Consider an increment in the external field $H(t)$. Typically, a spin flips ($S_i \to -S_i$) because of a change in sign of the local field: $h_i = J\Sigma_{j\in L_i} S_j + H(t) + H_i$, where L_i refers to the nearest neighbors of site i. This spin flip modifies the local fields at other sites, leading to an avalanche which stops when no further spins remain to be flipped. These avalanches are associated with Barkhausen noise observed in magnetic materials [6,7].

(3) Sethna *et al.* made a detailed study of the avalanche statistics. Let σ_H^2 denote the variance of the Gaussian random field in units of J^2. For weak disorder with $\sigma_H < \sigma_c$, the hysteresis loop is discontinuous at some field value $H(\sigma_H)$ due to an infinite-sized avalanche. This is a remnant of the rectangular loop arising for $\sigma_H=0$, corresponding to the pure ferromagnet. The infinite avalanche survives upto $\sigma_H=\sigma_c$, and we denote $H(\sigma_c)=H_c$. For $\sigma_H > \sigma_c$, the hysteresis loop is continuous and is characterized by small avalanches.

(4) The point (σ_c, H_c) is a critical point with a power-law distribution of the avalanche size s, $P(s,\sigma_c,H_c) \sim s^{-\tau}$, where $\sigma_c \simeq 2.23J$, $\tau \simeq 1.6$ for the $d=3$ RFIM. There are corrections to scaling for (σ, H) near (σ_c, H_c), and the corresponding $P(s)$ exhibit power-law behavior up to a cutoff scale which diverges as $(\sigma,H) \to (\sigma_c, H_c)$. Analogous statements hold for the distribution of avalanche durations [12].

Next, consider the case of the random-bond Ising model (RBIM), where $H_i=0$ in Eq. (2). An early study of hysteresis in the nearest-neighbor RBIM with an adiabatic magnetic field is due to Vives and Planes [13]. These authors undertook $d=2$ MC simulations of this model with a Gaussian J_{ij} distribution:

$$P(J_{ij}) = \frac{1}{\sqrt{2\pi J^2}} \exp\left[-\frac{(J_{ij}-J_0)^2}{2J^2}\right], \tag{3}$$

with average J_0 and variance J^2. They found that the RBIM does not exhibit the RPM property. This is because reverse spin flips may occur during an avalanche as there are also antiferromagnetic ($J_{ij}<0$) bonds in the system. These reverse flips destroy the partial ordering of metastable states which results in the RPM property. Vives and Planes also studied the avalanche distributions averaged over the entire cycle, which we denote as $P_{\text{int}}(s,J)$. They found results analogous to those for the RFIM: (1) There is a critical value J_c (in units of J_0) where $P_{\text{int}}(s,J_c) \sim s^{-\theta}$, with $\theta \simeq 1.45$ for the $d=2$ RBIM. (2) For $J<J_c$ (weak disorder), $P_{\text{int}}(s,J)$ decays slower than a power law with $P_{\text{int}} \sim s^{-\theta} e^{\lambda s}$ ($\lambda > 0$) for small values of s. There is also a peak corresponding to an infinite avalanche. On the other hand, for $J > J_c$ (strong disorder), $P_{\text{int}}(s,J)$ decays exponentially with s as $P_{\text{int}} \sim s^{-\theta} e^{\lambda s}$ ($\lambda<0$). Subsequently, Vives *et al.* [14] studied $T=0$ hysteresis in a range of disordered spin models, and confirmed that the above scenario is rather universal.

Finally, let us consider hysteresis in spin glasses, e.g., the RBIM with Gaussian disorder and $J_0=0$ in Eq. (3). The nearest-neighbor spin glass or the Edwards-Anderson model [4] (with $J_0=0$) corresponds to the strong-disorder limit in the Vives-Planes study [13]. Thus we expect the avalanche distributions to decay exponentially in this case. It is also relevant to study hysteresis in the SK spin glass model where all spins interact with each other, and one might expect a mean-field approach to be useful. In early work, Soukolis *et al.* [15] demonstrated the existence of hysteresis in the SK model by numerically studying the free-energy landscape.

In more recent work, Pazmandi *et al.* [16] (PZZ) studied $T=0$ hysteresis in the SK model with an adiabatic magnetic field. PZZ argued that the SK model exhibits (self-organized) critical behavior, $P(s,H) \sim s^{-\tau}$ with $\tau \simeq 1.0$, everywhere along the hysteresis loop. This should be contrasted with the RFIM and RBIM, where power-law behavior arises at a critical point in parameter space. We can paraphrase the PZZ argument as follows. A spin flip $S_j \to -S_j$ changes the local field at site i by $2J_{ij} \sim N^{-1/2}$. Therefore an avalanche occurs when the external field is increased by $\Delta_{\text{PZZ}} \sim N^{-1/2}$. Let $\Delta m = \langle s \rangle / N$ denote the average change in the magnetization due to the avalanche. Then, $dm/dH \sim (\langle s\rangle/N) N^{1/2} = N^{-1/2}\langle s \rangle$. The numerical results of PZZ showed that dm/dH is smooth everywhere, i.e., there is no system-sized avalanche. This occurs when $\langle s \rangle \sim N^{1/2}$, so the scale of the distribution for all H values is set by the system size rather than a tunable parameter. PZZ interpret this as a signature of *self-organized criticality* (SOC).

Finally, we mention two recent works in this context. Katzgraber *et al.* [17] have studied *reversal-field memory* effects in (a) $T=0$ simulations of the nearest-neighbor spin glass, and (b) experiments on thin films containing Co-Fe_2O_3 particles. Further, Deutsch and Narayan [18] demonstrated that multiple cycles are often necessary for loop closure in nearest-neighbor spin glasses if the external field is cycled between two moderate values, i.e., the system is not driven to saturation.

III. DETAILED RESULTS

As stated earlier, we focus on $T=0$ hysteresis in the SK model here. However, our protocol for changing the magnetic field is different from that of PZZ [16]. In our study, the magnetic field is incremented by a fixed amount Δ, in analogy with experiments. Then, the natural response variable is the magnetization jump Δm, which consists of multiple or single avalanches, depending on the value of Δ. We investigate the variation of the distribution $P(\Delta m, H)$ along the hysteresis loop. In the limit $N \to \infty$, we expect $P(\Delta m, H)$ to approach a delta function. However, for finite systems, we find that there is a strong H dependence which can be understood quantitatively in the weak-disorder limit.

Let us now provide details of our MC simulations. We study the SK Hamiltonian with spin-flip kinetics. The exchange coupling obeys the Gaussian distribution in Eq. (3) with $J_0=0$ and variance $\tilde{J}=J/N^{1/2}$. At $T=0$, the energy scale is set by J. We will subsequently set $J=1$, and all relevant quantities will be measured in units of J. The initial state of the system corresponds to $S_i(0)=-1 \;\forall\; i$ as $H(0)=-\infty$. The field is increased from $H \to H+\Delta$, and the following $T=0$ MC procedure is implemented. After each field increment, we calculate the local fields: $h_i=\Sigma_j J_{ij} S_j + H(t)$. A randomly chosen unstable spin with $h_i S_i < 0$ is flipped to $-S_i$. Depending on the value of Δ, there may be a number of unstable spins. This spin flip changes the local field at each site, thereby affecting the stability of other spins. The local fields are computed again and another spin with $h_i S_i < 0$ is flipped. This process is continued till there are no sites remaining with $h_i S_i < 0$. We designate the old and new configurations as $\{S_i\}$ and $\{S_i'\}$, respectively. There are three relevant physical quantities: (1) The *magnetization jump*, $\Delta m = \Sigma_i (S_i' - S_i)/N$. (2) The total number of spin flips in either direction as a fraction of the system size, which is denoted as n. (3) The *jump duration*, or the number of MC steps (MCS) before the system equilibrates in the new field.

The hysteresis loop can be closed by repeating the above procedure in the opposite direction, i.e., $H=\infty \to -\infty$. Our MC results were obtained for systems with $N=1000$ spins. We have confirmed that the hysteresis loops and their properties remain unchanged for larger values of N. The data presented here is obtained as an average over 20 hysteresis cycles for 800 disorder configurations. This statistics proves adequate to obtain smooth numerical results, as we demonstrate shortly.

Before proceeding, we compare our MC procedure with that of PZZ [16]. In the PZZ study, the magnetic field was changed adiabatically, i.e., the magnetic field was increased (or decreased) until the *least stable* spin flipped. This results in a spin-flip avalanche, and the statistics of these avalanches displays SOC, as discussed earlier. The typical field increment required to trigger an avalanche is $\Delta_{\mathrm{PZZ}} \sim N^{-1/2}$. In our study, the fixed parameter Δ may be either comparable to or much larger than Δ_{PZZ}. Hence we use the term *magnetization jump* rather than *avalanche* to characterize the system evolution. For $\Delta \gg \Delta_{\mathrm{PZZ}}$, we measure the averaged statistics of $N_a \sim \Delta/\Delta_{\mathrm{PZZ}}$ multiple interacting avalanches. This averaging implies that the distributions of these jumps can be understood in the framework of an *independent spin approximation*. On the other hand, for $\Delta \sim \Delta_{\mathrm{PZZ}}$, our results are analogous to those of PZZ, and earlier results by Bertotti and Pasquale [19]. Thus this study is complementary to the work of PZZ.

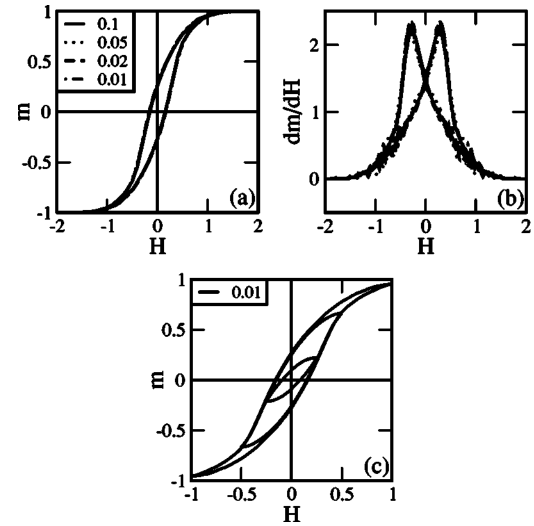

FIG. 1. (a) Hysteresis loop (m vs H) obtained from $T=0$ MC simulations of the SK spin glass in a magnetic field. Details of the simulation are provided in the text. The figure superposes data for $\Delta=0.1, 0.05, 0.02, 0.01$, as denoted by the specified line types. (b) Plot of dm/dH vs H for the data shown in (a). (c) Hysteresis subloops when the magnetic field is reversed at $H=0.5, 0.25$. The increment size is $\Delta=0.01$.

Let us first study the hysteresis loops. In the zero-disorder (infinite Δ) limit, the spins are uncoupled and the corresponding hysteresis loop is a step function. As the disorder amplitude is increased (or Δ is decreased), the loop opens out due to trapping in local metastable states, which differ from each other by small groups of spins. There is a distribution of barrier heights separating these minima. Typically, configurations in the free-energy landscape differ by clusters of $O(N^{1/2})$ spins, with an associated barrier energy $E_{\mathrm{dis}} \sim N^{1/4}$ in the SK model [5]. The MC simulations of Mackenzie and Young [20] demonstrate that the barrier distribution is approximately uniform up to this level. In the presence of a constant magnetic field H, the energy associated with a spin cluster is $E_{\mathrm{mag}} \sim N^{1/4} H$, so that $E_{\mathrm{mag}}/E_{\mathrm{dis}} \sim H$.

In Fig. 1(a), we plot m vs H for $\Delta=0.1, 0.05, 0.02, 0.01$ — as expected, the hysteresis loops superpose. The loops are smooth because of the averaging procedure. Each individual realization is characterized by irregular steps which correspond to discrete jumps. In Fig. 1(b), we plot dm/dH vs H for the data shown in Fig. 1(a). This plot quantifies the average magnetization jump at different field values. Finally, in Fig. 1(c), we plot m vs H again for $\Delta=0.01$. However, in this case, the fields are reversed at $H=0.5$, 0.25, and the loops are seen to exhibit the RPM property.

At this stage, it is relevant to query whether the shape of the hysteresis loops can be understood in the framework of mean-field (MF) theory. For the RFIM with long-ranged exchange interactions, there is an elegant MF argument due to Sethna *et al.* [12] which yields the loop shape and the critical exponents. The situation is far more difficult for the SK spin glass, as it is known that naive MF theory is not valid below the Almeida-Thouless line in the (T,H) plane [5]. The naive MF prediction for the local-field distribution in the SK spin glass has the Gaussian form [21]:

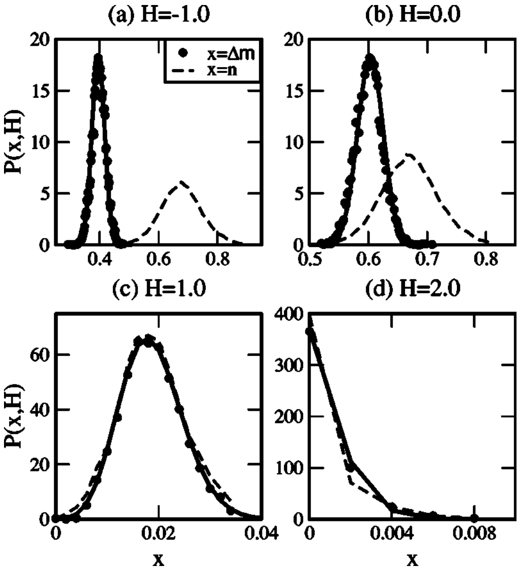

FIG. 2. Probability distributions, $P(x,H)$ vs x with $x=\Delta m$ or n, for the field increment $H \to H+\Delta$. The system size was $N=1000$, and $\Delta=1.0$. The solid line superposed on the data sets for $P(\Delta m,H)$ is the binomial distribution in Eq. (8) with p obtained from $\langle \Delta m \rangle$ for the numerical data. The different frames correspond to (a) $H=-1.0$, $p=0.395$; (b) $H=0.0$, $p=0.603$; (c) $H=1.0$, $p=0.019$; (d) $H=2.0$, $p=0.001$.

$$\tilde{P}(h_i,H) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(h_i-H)^2}{2\sigma^2}\right], \qquad (4)$$

with variance $\sigma^2 = qJ^2$. Here, the Edwards-Anderson order parameter q is obtained from $[\beta=(k_BT)^{-1}]$

$$q = \int_{-\infty}^{\infty} \frac{dz}{\sqrt{2\pi}} e^{-z^2/2} \tanh^2[\beta(H + J\sqrt{q}z)]. \qquad (5)$$

However, this Gaussian distribution is not even qualitatively reasonable at $T=0$. This is due to *replica-symmetry breaking* and has been studied extensively [5]. There are arguments and simulations [22–24] which demonstrate that $\tilde{P}(h_i,H)$ develops nonanalytic behavior at $h_i=0$. An empirical form for $\tilde{P}(h_i,H)$ at $T=0$ is obtained as follows [23,24]:

$$\tilde{P}(h_i,H) \simeq \frac{f|h_i|}{2\sigma(H)^2}\exp\left[-\frac{(h_i-H)^2}{2\sigma(H)^2}\right],$$

$$f^{-1} = \exp\left(-\frac{H^2}{2\sigma^2}\right) + \frac{\sqrt{2}H}{\sigma}\int_0^{H/(\sqrt{2}\sigma)} dx e^{-x^2}. \qquad (6)$$

Here, the scale of $\sigma(H)$ is set by J, and $\sigma(0) \simeq 1.28$ [22]. The functional form of $\sigma(H)$ is unknown and its dependence on H and the order parameters are necessary inputs to compute the hysteresis loop. Pazmandi *et al.* [16] have used heuristic arguments to obtain weak bounds on the outer limit of the hysteresis loop.

Next, let us discuss the magnetization-jump distributions at different points along the loop. In Figs. 2(a)–2(d), we plot MC data for $P(x,H)$ vs x (with $x=\Delta m$ or n) with four different field strengths, $H=-1.0,0.0,1.0,2.0$. Here, the distributions correspond to the step $H \to H+\Delta$, where $\Delta=1.0 \geqslant \Delta_{\mathrm{PZZ}} \sim 0.03$ for $N=1000$. The distributions in Figs. 2(a)–2(d) show a strong H dependence, with a crossover from a bell-shaped function to a monotonically decaying function. The distributions $P(\Delta m,H)$ and $P(n,H)$ show a similar behavior with $\langle \Delta m \rangle < \langle n \rangle$, as expected. For small values of $\langle \Delta m \rangle$ or $\langle n \rangle$, as in Figs. 2(c) and 2(d), the data sets for $P(\Delta m,H)$ and $P(n,H)$ superpose. The nature of these distributions depends on H, Δ, and the system size N. This can be understood by the arguments presented below for $P(\Delta m,H)$.

When the field is increased from $H \to H+\Delta$, the probability of the local field h_i changing sign from $h_i<0$ to $h_i>0$ is

$$p = \int_0^{\infty} dh_i[\tilde{P}(h_i,H+\Delta) - \tilde{P}(h_i,H)] \equiv p(H,\Delta). \qquad (7)$$

In the strong-field limit, the magnetization jump consists of multiple interacting avalanches and it is reasonable to assume that the spins are approximately independent. Then, the probability of $N\Delta m$ spins changing sign is obtained from the binomial distribution as

$$P(\Delta m) = \frac{N!}{(N\Delta m)!\,N(1-\Delta m)!} p^{N\Delta m}(1-p)^{N(1-\Delta m)}, \qquad (8)$$

with $\langle \Delta m \rangle = p$. The distributions $P(\Delta m,H)$ in Fig. 2 are fitted to this functional form, with p being determined from the numerical data. It is seen that Eq. (8) provides a good description of $P(\Delta m,H)$ at strong field amplitudes, where the independent-spin approximation applies.

It is useful to approximate the binomial distribution for large N as follows [25]:

(a) *Gaussian limit*: This arises for $Np \to \infty$ as $N \to \infty$ (p nonzero). Then, we obtain the Gaussian distribution:

$$P(\Delta m) \simeq \sqrt{\frac{N}{2\pi p(1-p)}}\exp\left[-\frac{N(\Delta m - p)^2}{2p(1-p)}\right]. \qquad (9)$$

The variance $p(1-p)/N \to 0$ as $N \to \infty$ and the distribution becomes sharply peaked at $\Delta m = p$.

(b) *Poisson limit*: This arises for Np finite as $N \to \infty$ ($p \to 0$). Then, Eq. (8) becomes the Poisson distribution:

$$P(\Delta m) \simeq \frac{(Np)^{N\Delta m}}{(N\Delta m)!} e^{-Np}. \qquad (10)$$

In a simulation involving a finite number of spins N, one can realize both the limiting cases in Eqs. (9) and (10), depending on the values of H and Δ. This is demonstrated by the data in Fig. 2.

The independent-spin approximation breaks down at weak fields (or small Δ), where magnetization jumps arise due to single avalanches. In that case, Eq. (8) is not a reasonable description of the Δm statistics. This is demonstrated in Fig. 3, where we plot $P(x,H)$ vs x for $H=0 \to \Delta$ with $\Delta=1.0,0.5,0.2,0.1,0.05,0.025$. We see that Eq. (8) fits $P(\Delta m,H)$ rather well for $\Delta=1.0$, 0.5. However, for $\Delta=0.2$, the binomial distribution is too sharply peaked in comparison with the numerical data. The disagreement is even stronger for smaller values of Δ, and we do not plot Eq. (8) in Figs.

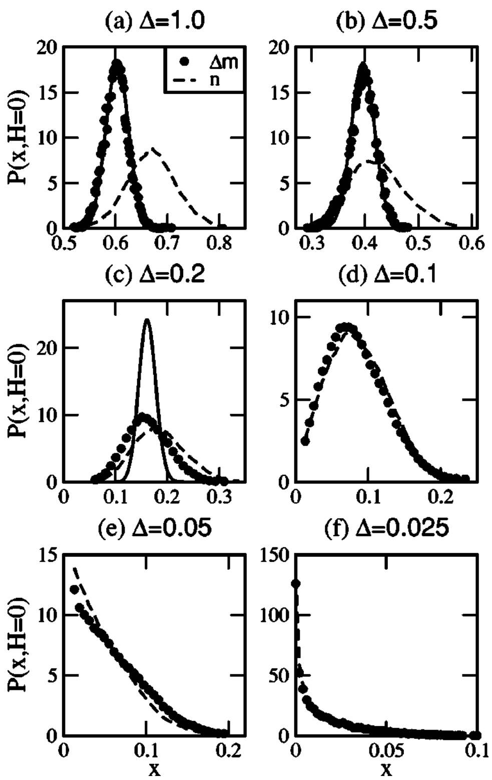

FIG. 3. Analogous to Fig. 2, except that the value $H=0$ is fixed and Δ is varied. The different frames correspond to (a) $\Delta=1.0$, $p=0.603$; (b) $\Delta=0.5$, $p=0.392$; (c) $\Delta=0.2$, $p=0.162$; (d) $\Delta=0.1$, $p=0.086$; (e) $\Delta=0.05$, $p=0.051$; (f) $\Delta=0.025$, $p=0.015$.

3(d)–3(f). The plots in Figs. 3(e) and 3(f) correspond to $\Delta \sim \Delta_{\mathrm{PZZ}} \sim 0.03$, and are consistent with power-law decay over a limited range. More generally, the crossover seen in Figs. 3(a)–3(f) with decrease in Np is qualitatively consistent with the above scenario.

IV. SUMMARY AND DISCUSSION

To summarize, we have studied hysteresis and magnetization-jump distributions for the $T=0$ dynamics of the SK spin glass in a time-dependent magnetic field. The field is incremented in equal steps, and is kept unchanged until the system equilibrates at the new field value. Therefore hysteresis results from trapping in local metastable states rather than a competition between the timescales of field and spin dynamics.

Our MC simulations show that the hysteresis loop exhibits return-point memory, which characterizes loops in the RFIM [12] though not in the nearest-neighbor RBIM [13]. We have also studied the magnetization-jump distribution functions $P(\Delta m, H)$ [and $P(n,H)$] along the hysteresis loop. For finite systems, these functions exhibit a strong dependence on the H value and the size of the field increment Δ. The relevant parameter is Np, where N is the system size and p is the spin-flip probability when the magnetic field is changed from $H \to H+\Delta$. For $\Delta \gg \Delta_{\mathrm{PZZ}} \sim N^{-1/2}$, the magnetization jumps are averages over multiple interacting avalanches. In that case, the independent-spin approximation is valid, and the H-dependent distributions are described by the binomial distribution. The plots of $P(\Delta m, H)$ vs Δm exhibit a crossover from a bell-shaped function to a monotonically decaying function as Np is decreased. However, the binomial distribution does not provide a reasonable description of the numerical data for $\Delta \sim \Delta_{\mathrm{PZZ}}$, due to a breakdown of the independent-spin approximation. In this limit, the SOC description of Pazmandi *et al.* [16] applies.

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